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On the origin of three generation free fermionic superstring models*

Alon E. Faraggi†‡

Department of Physics, University of Florida
Gainesville, FL 32611

ABSTRACT

The three generation superstring models in the free fermionic models have had remarkable success in describing the real–world. The most explored models use the NAHE set to obtain three generations and to separate the hidden and observable sectors. It is of course well known that the full NAHE set is not required in order to construct three generation free fermionic models. I argue that all the semi–realistic free fermionic models that have been constructed to date correspond to $Z_2 \times Z_2$ orbifolds. Thus, the successes of the semi–realistic free fermionic models, if taken seriously, suggest that the true string vacuum is a $Z_2 \times Z_2$ orbifold with nontrivial background fields and quantized Wilson lines.

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‡ e–mail address: faraggi@phys.ufl.edu

1. Introduction

The realistic free fermionic superstring models have had remarkable success in accounting for the observed low energy physics [1]. Not only do these models give rise to three chiral generations with the correct quantum numbers under the Standard Model gauge group, but perhaps more impressive is their success in providing plausible explanation to various properties of the observed low energy spectrum, like the stability of the proton [2] and the fermion mass hierarchy [3]. Perhaps the most outstanding success of the realistic free fermionic models is the correct prediction of the top quark mass [4], that was obtained in the context of these models several years prior to the experimental observation of the top quark by the CDF/D0 collaborations.

The successes of the realistic free fermionic superstring models may lead one to speculate that this indeed may be the path that nature has chosen. It should be emphasized that it is not claimed that one of the string models that were constructed to date is the string vacuum that nature has chosen. Indeed, such a claim will require far more elaborate analysis than has been performed to date. However, the remarkable successes of the free fermionic superstring models provide evidence that suggest that the eventually emerging true string vacua will share some of the basic underlying features of the realistic free fermionic models.

Taking this point of view, it is then of extreme importance to try to extract what are the basic underlying features of the realistic free fermionic models. One of the common properties of the realistic free fermionic superstring models is the fact that they all have three chiral generations.

It is well known that in the bosonic formulation of compactification of the heterotic string to four dimensions the number of generations is related to the Euler characteristic of the compactified manifold [5,6]. The free fermionic formulation [7], however, is formulated at a fixed point in the compactified space and all the information on the geometry of the compactified manifold is lost. The bosonic formulation of the heterotic string has a great advantage over the fermionic formulation in the sense that we can continuously deform the parameters of the compactified space and connect between string vacua that in the fermionic formulation would appear as distinct models. Thus, an extremely important task is to try to determine what is the underlying compactification of the realistic free fermionic models.

In this talk I argue that the underlying compactification of all the three generation free fermionic superstring models (that have been constructed to date) is $Z_2 \times Z_2$ orbifold compactification. A very simple realization of this underlying geometry is achieved with the so called "NAHE" set. However, it is well known that the complete NAHE* set is not required for obtaining three generations free fermionic models [8]. I argue that also in the case of non-NAHE models, there is an underlying compactification of a $Z_2 \times Z_2$ orbifold. I propose that the successes of the realistic free fermionic models, if taken seriously, indicate that the true string vacuum is a $Z_2 \times Z_2$ orbifold with nontrivial background fields and quantized Wilson lines.

2. Three generation models with the NAHE set

In the free fermionic formulation of the heterotic string in four dimensions all the world–sheet degrees of freedom required to cancel the conformal anomaly are represented in terms of free fermions. For the left–movers one has the usual space–time fields X^μ, ψ^μ, $(\mu = 0, 1, 2, 3)$, and in addition the following eighteen real free fermion fields: χ^I, y^I, ω^I $(I = 1, \cdots, 6)$, transforming as the adjoint representation of $SU(2)^6$. A model in this construction is defined by a set of boundary condition basis vectors, which are constrained by the string consistency requirements. The basis vectors generate a finite additive group Ξ. The physical states in the Hilbert space, of a given sector $\alpha\epsilon\Xi$, are obtained by acting on the vacuum with bosonic, and fermionic operators. For a periodic complex fermion f, there are two degenerate vacua $|+\rangle, |-\rangle$, annihilated by the zero modes f_0 and ${f_0}^*$ and with

∗ This set was first constructed by Nanopoulos, Antoniadis, Hagelin and Ellis (NAHE) in the construction of the flipped $SU(5)$. *nahe*=pretty, in Hebrew.

fermion numbers $F(f) = 0, -1$, respectively. The physical spectrum is obtained by applying the generalized GSO projections.

The basis vectors of the NAHE set are $\{\mathbf{1}, S, b_1, b_2, b_3\}$ with a choice of generalized GSO projections [11]. The sector S generates $N = 4$ space–time supersymmetry, which is broken to $N = 2$ and $N = 1$ space–time supersymmetry by b_1 and b_2, respectively. The gauge group after the NAHE set is $SO(10) \times E_8 \times SO(6)^3$. At the level of the NAHE set, each sector b_1, b_2 and b_3 give rise to 16 spinorial 16 of $SO(10)$. The Neveu-Schwarz sector produces massless states that transform as $(5 \oplus \bar{5})$ of $SO(10)$ and as singlets of $SO(10) \times E_8$.

The NAHE set divides the internal world–sheet fermions into several groups. The internal 44 right–moving fermionic states are divided in the following way: $\bar{\psi}^{1,\cdots,5}$ are complex and produce the observable $SO(10)$ symmetry; $\bar{\phi}^{1,\cdots,8}$ are complex and produce the hidden E_8 gauge group; $\{\bar{\eta}^1, \bar{y}^{3,\cdots,6}\}$, $\{\bar{\eta}^2, \bar{y}^{1,2}, \bar{\omega}^{5,6}\}$, $\{\bar{\eta}^3, \bar{\omega}^{1,\cdots,4}\}$ give rise to the three horizontal $SO(6)$ symmetries. The left–moving $\{y, \omega\}$ states are divided to, $\{y^{3,\cdots,6}\}$, $\{y^{1,2}, \omega^{5,6}\}$, $\{\omega^{1,\cdots,4}\}$. The left–moving $\chi^{12}, \chi^{34}, \chi^{56}$ states carry the supersymmetry charges.

An important consequence of the NAHE set is observed by extending the $SO(10)$ symmetry to E_6. Adding to the NAHE set a vector X with periodic boundary conditions for the set $\{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}\}$, extends the gauge symmetry to $E_6 \times U(1)^2 \times SO(4)^3$. The sectors $(b_j; b_j + X)$, $(j = 1, 2, 3)$ each give eight 27 of E_6. The $(NS; NS + X)$ sector gives in addition to the vector bosons and spin two states, three copies of scalar representations in $27 + \bar{27}$ of E_6.

In this model the fermionic states which count the multiplets of E_6 are the internal fermions $\{y, w|\bar{y}, \bar{\omega}\}$. The vacuum of the sectors b_j contains twelve periodic fermions. Each periodic fermion gives rise to a two dimensional degenerate vacuum $|+\rangle$ and $|-\rangle$ with fermion numbers 0 and -1, respectively. After applying the GSO projections, we can write the vacuum of the sector b_1 in combinatorial form

$$\left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4}\right] \left\{ \binom{2}{0} \left[\binom{5}{0} + \binom{5}{2} + \binom{5}{4}\right] \binom{1}{0}\right.$$

$$+\begin{pmatrix}2\\2\end{pmatrix}\left[\begin{pmatrix}5\\1\end{pmatrix}+\begin{pmatrix}5\\3\end{pmatrix}+\begin{pmatrix}5\\5\end{pmatrix}\right]\begin{pmatrix}1\\1\end{pmatrix}\Bigg\} \quad (1)$$

where $4=\{y^3y^4, y^5y^6, \bar{y}^3\bar{y}^4, \bar{y}^5\bar{y}^6\}$, $2=\{\psi^\mu, \chi^{12}\}$, $5=\{\bar{\psi}^{1,\cdots,5}\}$ and $1=\{\bar{\eta}^1\}$. The combinatorial factor counts the number of $|-\rangle$ in a given state. The two terms in the curly brackets correspond to the two components of a Weyl spinor. The $10+1$ in the 27 of E_6 are obtained from the sector b_j+X. The states which count the multiplicities of E_6 are the internal fermionic states $\{y^{3,\cdots,6}|\bar{y}^{3,\cdots,6}\}$. A similar result is obtained for the sectors b_2 and b_3 with $\{y^{1,2},\omega^{5,6}|\bar{y}^{1,2},\bar{\omega}^{5,6}\}$ and $\{\omega^{1,\cdots,4}|\bar{\omega}^{1,\cdots,4}\}$ respectively, which suggests that these twelve states correspond to a six dimensional compactified orbifold with Euler characteristic equal to 48.

The same model is generated in the orbifold language by moding out an $SO(12)$ lattice by a $Z_2\times Z_2$ discrete symmetry with standard embedding [9]. The $SO(12)$ lattice is obtained for special values of the metric and antisymmetric tensor and at a fixed point in compactification space. The metric is the Cartan matrix of $SO(12)$ and the antisymmetric tensor is given by $b_{ij}=g_{ij}$ for $i>j$. The sectors b_1, b_2 and b_3 correspond to the three twisted sectors in the orbifold models and the Neveu–Schwarz sector corresponds to the untwisted sector.

The reduction to three generations is illustrated in table 1. In the realistic free fermionic models the vector X is replaced by the vector 2γ in which $\{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\}$ are periodic. This reflects the fact that these models have (2,0) rather than (2,2) world-sheet supersymmetry. At the level of the NAHE set we have 48 generations. One half of the generations is projected because of the vector 2γ. Each of the three vectors in table 1 acts nontrivially on the degenerate vacuum of the fermionic states $\{y,\omega|\bar{y},\bar{\omega}\}$ that are periodic in the sectors b_1, b_2 and b_3 and reduces the combinatorial factor of Eq. (1) by a half. Thus, we obtain one generation from each sector b_1, b_2 and b_3.

2. Three generation models without the NAHE set

In the previous section we saw how three generation free fermionic models are obtained if the basis contains the full NAHE set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$. In these models the connection with the $Z_2 \times Z_2$ orbifold is readily established. The basis vector b_3 is replaced with the basis vector $\xi_1 = 1 + b_1 + b_2 + b_3$. The set $\{\mathbf{1}, S, \xi_1\}$ generates a toroidal compactified model with $N = 4$ supersymmetry and $SO(28) \times E_8$ gauge group. The boundary condition vectors b_1 and b_2 the correspond to the $Z_2 \times Z_2$ orbifold twist. The three sectors b_1, b_2 and $b_3 = \mathbf{1} + b_1 + b_2 + \xi_1$ then correspond to three twisted sectors of the $Z_2 \times Z_2$ orbifold model. The reduction to three generations is achieved by reducing the number of fixed points from each twisted sector to one, by adding three additional boundary condition basis vectors. Each subsequent boundary condition basis vector reduce the number of fixed points from each sector b_1, b_2 and b_3 by a factor of two. At the same time the gauge group is broken to one of the maximal subgroups of $SO(10)$. Many other desirable phenomenological properties, like doublet–triplet splitting, can be achieved for appropriate assignment of the boundary conditions in the additional boundary condition basis vectors.

The free fermionic formulation is formulated at a point in the moduli space that may be prefered from a dynamical point of view. The reason being that the free fermionic formulation is formulated near the self–dual point in the compactification space. Studies of the effective moduli potential within the context of dynamical supersymmetry breaking by gaugino condensates, indeed suggest that moduli VEVS of the order of the self–dual radius minimize the effective moduli potential [10]. The structure exhibited by the NAHE set is then seen to be very robust in getting three generation models with very desirable phenomenological properties.

These properties of the realistic free fermionic models suggest that it may be that the true string vacuum is in the close vicinity of these models. An extremely important question is then to ask what are the properties of these models that are truly model independent. While it may be naive to expect that one specific string model will turn out to be the true string vacuum, it is reasonable to expect that perhaps we will be able to guess the correct string compactification, or the correct neighborhood of the true string vacuum.

In the language of the free fermionic models the low energy phenomenological properties are related to the choices of boundary condition basis vectors and generalized GSO phases. We can then ask which choices of basis vectors and GSO phases, in this limited class of models, are necessary to obtain certain desirable phenomenological properties. One such phenomenological criteria is the requirement of three chiral generations.

While the NAHE set provides an elegant way for obtaining three chiral generations, it is of course well known that the full NAHE set is not required for constructing three generation free fermionic models. The first published example of such a model was published in Ref. [8]. This model is shown in table 3 (with a slight change of notation). In this model the sectors that may produce chiral generations are the sectors b_1, b_2 and b_3. The degenerate vacuum of the fermionic zero modes can be represented similar to Eq. (1). The internal fermionic states can also be divided into the same groups as with the NAHE set. Each sector b_j then gives rise to 16 generations. This number is then reduced by the choices of boundary conditions in the remaining boundary condition basis vectors. It is easy to see from table 2 that in this model the sector b_1 and b_2 give rise to one and two generations respectively. From the table we observe that the degeneracy of the vacuum due to the real fermions in the sector b_1 is removed completely, while in the sector b_2 a double degeneracy remains. The sector b_3 does not obey the chirality condition of Ref. [11] and therefore gives rise only to non-chiral matter. The chirality condition of Ref. [11] states that to obtain from a given sector, b_j, chiral 16 representation of $SO(10)$ we need a second vector, b_k, with $\{\psi^\mu, \bar{\psi}_{1\cdots5}\}$ periodic in both vectors and the intersection between the remaining boundary conditions is empty. If this condition is not satisfied then, as long as the $SO(10)$ symmetry is not broken, a given vector b_j will give an equal number of 16 and $\bar{1}6$ and thus will not contribute to the net number of generations. The vector b_k is not necessarily a basis vectors but must exist as a combination of basis vectors. Therefore in this model the sector b_3 produces one 16 and one $\overline{16}$ representation of $SO(10)$.

This model is an example how three generation can be obtained without the full NAHE set. Of course, there may exist many other possibilities. For example, as shown in Ref. [8] it is possible to add the vector α with $\{y^5, \omega^5, y^6, \omega^6, |\bar{y}^5, \bar{\omega}^5, \bar{y}^6, \bar{\omega}^6, \bar{\eta}^1, \bar{\phi}^{6,\cdots,8}\}$ periodic and the remaining boundary conditions antiperiodic to the basis of table 3. With this basis vector, the combination $b_1 + \alpha$ projects the $\overline{16}$ from the sector b_3 and reduces the double degeneracy of the generations from the sector b_2. Thus, with basis vector α this model contains one generation from each sector b_1, b_2 and b_3.

The important question, however, is whether all these three generation models possess some common structure.

The answer, of course, is that all these models are related to $Z_2 \times Z_2$ orbifolds. The orbifold moding however does not act on a torodially compactified model with $N = 4$ space–time supersymmetry and $SO(12) \times E_8 \times E_8$ or $SO(12) \times SO(16) \times SO(16)$ gauge group, as in the case of the models that contain the full NAHE set. In the non–NAHE models, the sectors ξ_1 and ξ_2 that produce the spinorial of $SO(16)$ in the observable and hidden E_8s are not present, therefore the space–time gauge group of the torodially compactified model is $SO(44)$. This gauge symmetry is obtained in the bosonic formulation for appropriate choices of the metric, antisymmetric tensor and Wilson lines [12].

In the Kagan–Samuel model above the basis vectors $\{\mathbf{1}, S\}$ generates a torodially compactified model with $N = 4$ and $SO(44)$ gauge group. The basis vectors b_1 and b_2 correspond to the $Z_2 \times Z_2$ orbifold twisting. The third twisted sector of the orbifold model is still present, and is the combination $\mathbf{1} + b_1 + b_2$. In this model the vector ξ_2 which generates the spinorial representation of $SO(16)$ in the adjoint representation of the hidden E_8 gauge group is absent. Consequently, the states from the third twisted sector, $\mathbf{1} + b_1 + b_2$, are not massless states.

All the three generation free fermionic models that have been constructed to date contain the basis vectors $\{\mathbf{1}, S\}$. The basis vector S produces $N = 4$ space–

time supersymmetry. The only way to reduce the number of supersymmetry generators that are produced by this basis vector, is by moding by a $Z_2 \times Z_2$ twist. Therefore, the connection between the three generation free fermionic models and $Z_2 \times Z_2$ orbifold compactification is in fact expected. By free fermions we confine ourselves to conformal field theory blocks that are either complex fermions or Ising model operators. Recently, it was found that three generation models can also be obtained if one relaxes this constraint [13]. In the models of Ref. [13], the vector S is used to generate the space–time supersymmetry. Consequently, in this models, the $Z_2 \times Z_2$ structure is preserved in the supersymmetric sector, while it is relaxed in the bosonic sector. These models make use of more complicated conformal solutions that do not correspond to free fermions.

4. Conclusion

In this talk, I discussed the correspondence between three generation free fermionic models and orbifold compactification. All the three generation free fermionic models that have been constructed to date correspond to $Z_2 \times Z_2$ orbifold compactifications. Other compactifications may be constructed in the free fermionic formulation by using different supersymmetry generators [14] (It is of course possible that these SUSY generators can also lead to three generation models). If the successes of the realistic free fermionic models are to be taken seriously, then they indicate that the true string vacuum is related to a $Z_2 \times Z_2$ orbifold compactification with nontrivial background fields. Out of the plethora of orbifold compactifications [15], $Z_2 \times Z_2$ orbifolds, therefore, deserve special attention.

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	ψ^μ	$\{\chi^{12};\chi^{34};\chi^{56}\}$	$\bar\psi^1, \bar\psi^2, \bar\psi^3, \bar\psi^4, \bar\psi^5, \bar\eta^1, \bar\eta^2, \bar\eta^3$	$\bar\phi^1, \bar\phi^2, \bar\phi^3, \bar\phi^4, \bar\phi^5, \bar\phi^6, \bar\phi^7, \bar\phi^8$
α	0	$\{0,\ 0,\ 0\}$	1, 1, 1, 0, 0, 0 , 0, 0	1, 1, 1, 1, 0, 0, 0, 0
β	0	$\{0,\ 0,\ 0\}$	1, 1, 1, 0, 0, 0, 0, 0	1, 1, 1, 1, 0, 0, 0, 0
γ	0	$\{0,\ 0,\ 0\}$	$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$	$\frac{1}{2}$, 0, 1, 1, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, 0

	$y^3y^6, y^4\bar y^4, y^5\bar y^5, \bar y^3\bar y^6$	$y^1\omega^6, y^2\bar y^2, \omega^5\bar\omega^5, \bar y^1\bar\omega^6$	$\omega^1\omega^3, \omega^2\bar\omega^2, \omega^4\bar\omega^4, \bar\omega^1\bar\omega^3$
α	1, 0, 0, 0	0, 0, 1, 1	0, 0, 1, 1
β	0, 0, 1, 1	1, 0, 0, 0	0, 1, 0, 1
γ	0, 1, 0, 1	0, 1, 0, 1	1, 0, 0, 0

Table 1. A three generations model with the NAHE set. The choice of generalized GSO coefficients is: $c\begin{pmatrix} b_j \\ \alpha,\beta,\gamma \end{pmatrix} = -c\begin{pmatrix} \alpha \\ 1 \end{pmatrix} = c\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = -c\begin{pmatrix} \beta \\ 1 \end{pmatrix} = c\begin{pmatrix} \gamma \\ 1,\alpha \end{pmatrix} = -c\begin{pmatrix} \gamma \\ \beta \end{pmatrix} = -1$ (j=1,2,3), with the others specified by modular invariance and space–time supersymmetry.

	ψ^μ	$\{\chi^{12};\chi^{34};\chi^{56}\}$	$\bar{\psi}^1, \bar{\psi}^2, \bar{\psi}^3, \bar{\psi}^4, \bar{\psi}^5, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3$	$\bar{\phi}^1, \bar{\phi}^2, \bar{\phi}^3, \bar{\phi}^4, \bar{\phi}^5, \bar{\phi}^6, \bar{\phi}^7, \bar{\phi}^8$
1	1	{1, 1, 1}	1, 1, 1, 1, 1, 1, 1, 1	1, 1, 1, 1, 1, 1, 1, 1
S	1	{1, 1, 1}	0, 0, 0, 0, 0, 0, 0, 0	0, 0, 0, 0, 0, 0, 0, 0
b_1	1	{1, 0, 0}	1, 1, 1, 1, 1, 1, 0, 0	0, 0, 0, 0, 0, 0, 0, 0
b_2	1	{0, 1, 0}	1, 1, 1, 1, 1, 0, 1, 0	0, 0, 0, 0, 0, 0, 0, 0
b_3	1	{0, 1, 0}	1, 1, 1, 1, 1, 0, 1, 0	0, 0, 0, 0, 0, 0, 0, 0
P	0	{0, 0, 0}	0, 0, 0, 0, 0, 0, 0, 0	0, 0, 0, 0, 1, 1, 1, 1
α	0	{0, 0, 0}	$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$	$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, 0, 0, 1, 1

	$y^3y^4, y^5\bar{y}^5, y^6\bar{y}^6, \bar{y}^3\bar{y}^4$	$y^1\bar{y}^6, y^2\bar{y}^2, \omega^5\bar{\omega}^5, \bar{\omega}^6\bar{\omega}^6$	$\omega^1\bar{\omega}^1, \omega^2\bar{\omega}^2, \omega^3\bar{\omega}^3, \omega^4\bar{\omega}^4$
1	1, 1, 1, 1	1, 1, 1, 1	1, 1, 1, 1
S	0, 0, 0, 0	0, 0, 0, 0	0, 0, 0, 0
b_1	1, 1, 1, 1	0, 0, 0, 0	0, 0, 0, 0
b_2	0, 0, 0, 0	1, 1, 1, 1	0, 0, 0, 0
b_3	0, 1, 1, 0	0, 0, 0, 0	1, 1, 0, 0
P	0, 1, 0, 0	0, 0, 1, 0	0, 0, 0, 0
α	0, 0, 0, 1	1, 0, 0, 0	1, 0, 0, 0

Table 2. The three generations $SU(5)\times U(1)$ Kagan–Samuel model. The choice of GSO phases is: $c\begin{pmatrix} \mathrm{S} \\ \mathbf{1}, b_j \end{pmatrix} = -c\begin{pmatrix} \mathrm{S} \\ P,\alpha \end{pmatrix} = c\begin{pmatrix} b_1 \\ b_2, b_3 \end{pmatrix} = c\begin{pmatrix} b_2 \\ b_3 \end{pmatrix} = -c\begin{pmatrix} b_j \\ P \end{pmatrix} = -c\begin{pmatrix} b_1 \\ \alpha \end{pmatrix} = 1$ (j=1,2,3) and $c\begin{pmatrix} b_2, b_3 \\ \alpha \end{pmatrix} = i$, with the others specified by modular invariance and space–time supersymmetry (The notation and GSO phases may be different from KS). This model does not contain the full NAHE set.